The Exam consists of one page   Answer all questions   No. of questions: 5 Total Mark: 70

Question 1

Find $\mathbf{y}^{\grave{}}$ from the following:

(a) $y = 2 + 2^x + 2^{x^3}$ (b) $y = x^4 . \sinh x^4$ (c) $y = \log x . \tanh x$

(d) $y = \tan^{-1} x + \tanh^{-1} x$ (e) $y = t. \ln t ,\ x = t. e^t$ (f) $y^4 = x + \ln(x - y)$

[18]

Question 2

Find the following integrals:

(a) $\int (x^3 + \frac{1}{x^3} + 3^x) dx$ (b) $\int (\frac{x}{1+x^2} + \frac{1}{\sqrt{1+x^2}}) dx$ (c) $\int (2 - \sqrt{x})^2 \, dx$

(d) $\int (\frac{1}{x+1} + \frac{1}{1+x^2}) dx$ (e) $\int e^x (3 + e^x)^7 \, dx$ (f) $\int x \ln x \, dx$ (g) $\int (3 + \cosh x) \, dx$ (h) $\int \sin^4 x \, dx$ (i) $\int \frac{x+2}{x^2-4x+3} dx$

[18]

Question 3

(a) Find the area of the region between the curve $y = x^3 - 8$, x-axis, x in [1, 3].

[4]

(b) If the region between the curve $y = 1 + x^3$,   x-axis,   x in [1, 2] is rotated about (i) x-axis (ii) y-axis. Find the volume of the generated solids $V_x \ and \ V_y$.

[8]

(c) Find the length of the curve $y = \frac{2}{3} x \sqrt{x}$,   x in [1, 2].

[4]

Question 4

(a) State the definition of the plane.

[2]

(b) Find the angle between the planes : $x - 2y + 2z + 5 = 0$, $3x - 4z - 1 = 0$.

[3]

(c) Write the equation of the plane that passes through the points: (1, 3, 2), (−1, 0, 4), (2, 0, 4).

[5]

Question 5

(a) Find the angle between the lines :

$x = -3t + 1, \ \ y = 4t + 1, \ \ z = 3$ and $x = \frac{y-1}{2} = \frac{z+1}{-2}$

[2]

(b) Write the equation of the plane that passes through the point (0, −1, 3) and its normal vector $\overline{N} = 2i - 3j + k$.

[2]

(c) Write the name of each surface:

(i) $x^2 + y^2 + z^2 - 2x - 3 = 0$ (ii) $z^2 = x^2 + 3y^2$

(iii) $y^2 + 3z^2 = 4$ (iv) $x^2 + 2z^2 = y^2$

[4]

Good Luck   *Dr. Mohamed Eid*